Mathematics 290 **Exam 3** **Spring** 2007

April 3, 2007

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Name

Technology used: ____

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

The Problems

Do two (2) of these computational problems

1. Show that the subset $V = \{p(x) \in P_3 : \ p(1) = p(-1), p(2) = p(-2)\}$ is a subspace of P_3.
2. Find, with proof, a basis for the subspace $V = \{p(x) \in P_3 : \ p(1) = p(-1), p(2) = p(-2)\}$ of P_3.
3. Determine if the set $\left\{ \begin{bmatrix} -2 & 3 & 4 \\ -1 & 3 & -2 \end{bmatrix}, \begin{bmatrix} 4 & -2 & 2 \\ 0 & -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & -2 & -2 \\ 2 & 2 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & -2 \end{bmatrix}, \begin{bmatrix} -1 & 2 & -2 \\ 0 & -1 & -2 \end{bmatrix} \right\}$ is linearly independent in $M_{2,3}$.

Do two (2) of these problems from the text, class, old exams or homework

1. Suppose that W is a vector space with dimension 5, and U and V are subspaces of W, each of dimension 3. Prove that $U \cap V$ contains a non-zero vector. Be careful, do not assume that every basis of of U contains a vector in V.
2. Suppose that A is an invertible matrix. Prove that the matrix $\overline{(A^t)}$ is invertible and determine what that inverse is.
3. Do both of the following.
   (a) Prove that if V is a vector space and U and W are subspaces of V, then $U \cap W$ is a subspace of V.
   (b) Give an example of a specific vector space V and specific subspaces U, W where $U \cup W$ is **not** a subspace of V.
4. Prove that if A is a square matrix where $N(A^2) = N(A^3)$, then $N(A^4) = N(A^3)$. Here $N(A^2)$ denotes the null space of A^2.

Do two (2) of these less familiar problems

1. Suppose that A is a square matrix and there is a vector $\vec{b}$ such that $LS\left(A, \vec{b}\right)$ has a unique solution. Prove that A is nonsingular. Note that you **do not** know that $LS\left(A, \vec{b}\right)$ has a unique solution for every $\vec{b}$. You are only told that there is a unique solution for one particular $\vec{b}$.

2. Suppose that A is an $n \times n$ matrix and B is an $n \times p$ matrix. Show that the column space of AB is contained in the column space of A.

3. Let $\vec{v}$ be a particular vector in $\mathbf{C}^m$. Show that the set $V = \{\vec{w} \in \mathbf{C}^m : \vec{w} \text{ is orthogonal to } \vec{v}\} = \{\vec{w} \in \mathbf{C}^m : \langle \vec{w},\ \vec{v} \rangle = 0\}$ is a subspace of $\mathbf{C}^m$. The vector space V is called the orthogonal complement of the subspace of $\mathbf{C}^m$ spanned by $\{\vec{v}\}$.

4. If $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in \mathbf{C}^3$, find a basis for the orthogonal complement of the subspace of $\mathbf{C}^3$ spanned by $\{\vec{v}\}$. [See problem 3 immediately above this problem.]